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A review of baseflow recession analysis

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Abstract

Recession analysis is a well-known tool in hydrological analysis. Its application, however, poses many methodical questions, and throughout the literature numerous solutions have been sought. The quantification of the recession curve involves the selection of an analytical expression, derivation of a characteristic recession and optimization of the recession parameters. A major problem is the high variability encountered in the recession behaviour of individual segments. The segments represent different stages in the outflow process, and a physically based short-term or seasonal influence on the recession rate adds to the problem of deriving a characteristic recession. This paper discusses these elements of recession analysis and reviews different ways of characterizing the baseflow recession rate.

1. Introduction

Low flow characteristics of rivers have been increasingly utilized in recent years as the demand for water has increased. Information on low flow characteristics provides threshold values for different water-based activities and is required for such water resource management issues as water supply, irrigation, and water quality and quantity estimates. An understanding of the outflow process from groundwater or other delayed sources is also essential in studies of water budgets and catchment response.

1.1. Definitions

During dry weather, water stored in the catchment is removed by soil- and groundwater drainage and by evapotranspiration. These processes proceed at different rates in time and space, and are not readily quantified. The gradual depletion of discharge

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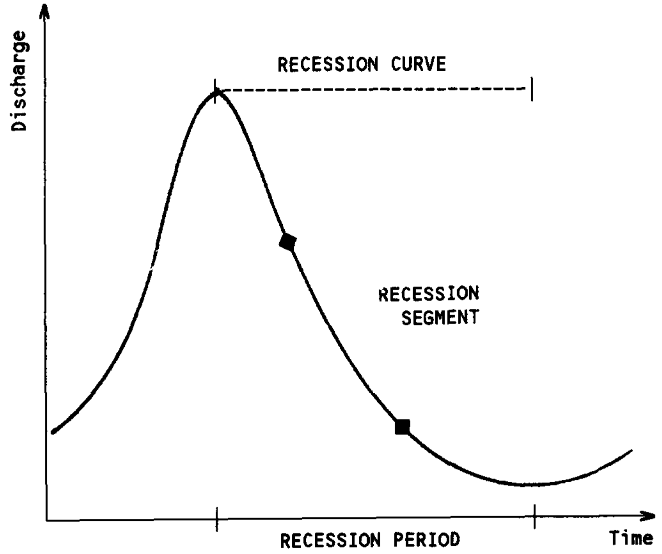

Fig. 1. Recession curve, period and segment.

during periods with little or no precipitation constitutes the drainage or recession rate, graphically presented as the recession curve (Fig. 1).

Recession analysis can be traced back as far as studies by Boussinesq in 1877 and Maillet in 1905. In many early works, recession analysis is associated with theoretical groundwater flow. As it is difficult to distinguish between the different sources of flow, this review considers methods that are concerned with baseflow in a wide context, including groundwater, unsaturated soil and lake drainage. In fact, as shown by a number of workers, notably Hewlett (1961), groundwater as traditionally defined is rarely present in small upland catchments. Unsaturated drainage is extremely complex both to model and measure.

Baseflow as defined by Hall (1968) is the portion of flow that comes from groundwater or other delayed sources. Hall gave in his work an excellent review of different aspects of baseflow recessions. Appleby (1970) added further comments to the recession and baseflow problem with special reference to the work by Hall. Hall stated that many of the difficulties of application originate in the mathematical assumptions and in problems of interpreting actual stream hydrographs. The quality of the low flow data is often a limiting factor when analysing hydrograph recessions. Accuracy and frequency of the flow measurements determine and restrict the processes that can be studied. Many shortcomings of recession analysis and the large number of methods that exist are also a result of the high variation in recession behaviour, both within and between catchments. This review reveals that many problems still remain in recession analysis. However, modern computers have allowed the development of automatic and objective methods, which eliminate some of the subjective elements of recession analysis and encourage a wider use.

1.2. Areas of application

The recession curve tells in a general way about the natural storages feeding the stream. Accordingly, it contains valuable information concerning storage properties and aquifer characteristics, and recession analysis has been useful in many areas of water resource planning and management: in low flow forecasting to benefit the management of irrigation, water supply, hydroelectric powerplants and waste dilution; in mathematical modelling for calibration of, or input to, rainfall–runoff models; in hydrograph analysis for graphical separation of different flow components; in frequency analysis for estimating low flow statistics; in regional low flow studies for indexing the storage capacity of the catchment.

Recession equations can easily be applied to forecast low flows in gauged rivers (Bako and Owoade, 1988). Today, low flow forecasting is commonly performed as an integrated part of real-time simulation of runoff using calibrated rainfall–runoff models. Recession behaviour and low flows are determined by the storage volumes and drainage functions selected in the model and the approaches are numerous (Franchini and Pacciani, 1991). Recession analysis can assist in determining the drainage parameters of the models (Harlin, 1991). Integrating the recession curve provides an estimate of storage, that is, water available for drainage (Knisel, 1963; Boughton and Freebairn, 1985; Korkmaz, 1990), and has similarly been applied to estimate the reservoir storage required to augment low flow (Kachroo, 1992).

Initial storage capacity prior to a storm was estimated by Troch et al. (1993) using the low flow analysis proposed by Brutsaert and Nieber (1977). The Brutsaert–Nieber procedure allows recession parameters to be related to aquifer characteristics following the Dupuit–Boussinesq theory. Troch et al. (1993) extended the model concept to the catchment scale to obtain a catchment-scale parameter representing initial storage, namely the effective depth to the water table.

Low flow frequency analyses are normally performed by fitting a hypothetical distribution function to the data. A physically based frequency estimate of low flows based on a probability analysis of hydrograph recessions was presented by Loganathan et al. (1986). Gottschalk and Perzyna (1989) derived a closed form of a physically based distribution function for low flows, where two out of four parameters were estimated from the baseflow recessions. By incorporating parameters of the recession process, the distribution functions can be estimated from short observation series. Perzyna (1990) found, however, that a minimum of 10 years was necessary to provide reliable estimates of the recession parameters. Recession analysis is also applicable when estimating low flow statistics using baseflow correlation procedures. This is of interest at sites with very short records, provided a nearby, hydrologically similar site with a long flow record exists (Hardison and Moss, 1972; Stedinger et al., 1993). At the ungauged site, Curran (1990) proposed a simple regression model between low flow and recession characteristics.

Regional regression models for low flows have developed for the purpose of estimating low flow statistics at the ungauged site from catchment characteristics. The influence of soil and geology is frequently reported as the major terrestrial influence on low flows; the problem is how to represent this influence numerically.

An indirect representation of the storage capacity by means of recession characteristics has been chosen in some multiple regression studies (Institute of Hydrology, 1980; Browne, 1981; Vogel and Kroll, 1991; Tallaksen, 1989; Demuth and Hagemann, 1993). Vogel and Kroll (1992) developed the regression equations by first formulating a conceptual watershed model for low flow by extending a simple stream–aquifer low flow model, similar to that of Brutsaert and Nieber (1977), to an entire catchment. The variables included in this model—catchment area, average slope and the recession constant—were then proved to be significant, independent variables in a regional regression model.

The recession curve holds substantial promise for use as an index of storage, provided a suitable way of estimation at the ungauged site exists. The relationship between geology and recession rate has frequently been reported, but has often, particularly in earlier works, been limited to a qualitative description (Ayers and Ding, 1967; Musiake et al., 1975). Average values of the recession constant have been derived for different physiographical regions or geological formations (Knisel, 1963; Wright, 1970; Trainer and Watkins, 1974; Browne, 1978; Ando et al., 1986; Bingham, 1986) and applied as an index of representativeness in hydrological regions (Vaugh, 1973). Quantitative relationships with catchment characteristics have been investigated by several workers (Farvolden, 1963; Andersen, 1972; Klaassen and Pilgrim, 1975; Tjomsland et al., 1978; Pereira and Keller, 1982b; Petras, 1986; Zecharias and Brutsaert, 1988; Demuth, 1989; Tallaksen, 1989; Curran, 1990). Although it has been possible to establish estimation equations for recession characteristics, these works often show that the recession characteristics are inadequately described by the indices available. The most important indices found to affect the recession rate in these studies are related to geological characteristics, relief and climate.

Recession analysis is also useful when estimating the storm hydrograph recession characteristics (Kelman, 1980; Fedora and Beschta, 1989), and has been applied to correct discharge measurements using dilution gauging on the storm recession (Gilman, 1977a,b). However, analysis of storm hydrograph recessions is generally handled by routing and hydraulic techniques and not discussed here.

1.3. Elements of recession analysis

It is necessary to derive a quantitative expression for the recession curve in order to incorporate it in comparative studies. This can be done in several ways, as also expressed in the literature. The quantification process raises the following questions: which analytical expression is preferable for the data in question? Which method should be used to derive a characteristic recession for a catchment? Which method should be used to optimize the recession parameters? How should one treat the high variability encountered in the recession behaviour of individual segments?

Recession analysis generally suffers from the fact that no satisfactory definition of a catchment characteristic recession exists. This is mainly due to the high time variability found in recessions. The result is a lack of consistency in obtaining recession characteristics which has limited a wider use. This paper discusses these issues separately and reviews different ways of characterizing the baseflow recession rate.

2. Analytical expressions

Recession analysis studies the outflow function:

$$Q = Q(t) \quad (1)$$

where Q is the rate of flow and t the time. There is no standard technique at present available for determining this relationship. Some workers have studied the recession flow from a theoretical point of view, starting with the basic flow equations, whereas others have studied empirical relationships. There are nearly as many methods as there are works on recession analysis, which makes it difficult to compare and evaluate the results. In this review, the most frequently used analytical expressions in recession analysis have been classified according to their modelling approach.

2.1. Modelling recession from basic flow equations

The basic nonlinear differential equation governing unsteady flow from a large unconfined aquifer to a stream channel was presented by Boussinesq in 1877 (Hall, 1968). The equation is valid under idealized conditions, that is, no evapotranspiration, leakage or recharge. Theoretical equations for groundwater flow derived from the Boussinesq equation have been presented by Singh (1968, 1969), and Brutsaert and Nieber (1977) gave a wide presentation of theoretical equations for aquifer drainage used during the past century, many of which can be derived directly from Boussinesq's work.

Based on theoretical equations for groundwater flow, the decay of the aquifer outflow rate can be modelled as a function of aquifer characteristics. For application, simplifying assumptions concerning physical properties of the aquifer have to be made, and these physically based equations are in general restricted to aquifers that are homogeneous, uniform, isotropic and confirmed by specific boundary restrictions. Relationships of this type have been presented and discussed among others by Rorabaugh (1964), Singh and Stall (1971), Daniel (1976), Brutsaert and Nieber (1977), Birtles (1978), Petras (1986), Zecharias and Brutsaert (1988), Vogel and Kroll (1992) and Troch et al. (1993). These works suggest that physical expressions of the recession rate might be successful at the catchment scale for relatively homogeneous conditions. Application in a heterogeneous catchment and at a regional scale is, however, limited.

Boussinesq linearized the flow equation by assuming a system which can be referred to as a Depuit–Boussinesq aquifer. This is a system where the Depuit assumptions of negligible vertical flow components are valid, and where the effect of capillarity above the water table can be neglected. A linearized Depuit–Boussinesq equation results in a simple exponential equation, given as (2) or in the alternative forms (2a) or (2b):

$$Q_t = Q_0 \exp(-t/C) \tag{2}$$

$$Q_t = Q_0 \exp(-a_1 t) \tag{2a}$$

$$Q_t = Q_0 k^t \tag{2b}$$

where Q_t is the flow at time t, Q_0 is the flow when $t = 0$ and C, a_1 and k are constants. The curve plots as a straight line of slope $\log k$ in a semilogarithmic plot of t against $\log Q_t$. C in (2) is the time elapsed between any discharge Q and Q/e of the recession. C is related to the 'half flow period' by the equation

$$C = -t_{0.5}/\ln\left(\frac{1}{2}\right) \tag{3}$$

where $t_{0.5}$ is the time required to halve the streamflow (Martin, 1973). Whereas C has the dimension of time, k is a dimensionless quantity whose value depends on the time unit chosen. C is related to k by the following expression:

$$C = -t/\ln k \tag{4}$$

where $t = 1$ represents one time unit. k in (2b) can take on values in the interval [0,1], but the time unit of interest will normally imply a k value in the upper range, commonly $k > 0.7$. If the recessions are characterized by very slow recession rates, a clustering at very high k values will result. A larger sensitivity in the equation constant for slow recessions can be achieved using Eq. (2) or (2a). If the recession constant is to be applied within statistical analysis, normally it has to meet special assumptions concerning its properties as a variable; for instance, that it has a normal distribution. It is important that these aspects of the recession constant are considered when necessary.

Horton (1933) suggested the nonlinear relationship

$$Q_t = Q_0 \exp(-a_2 t^m) \tag{5}$$

where a_2 and m are constants. This expression is often referred to as the Horton double exponential. The equation can be derived from Eq. (2) by a simple time transformation (Hall, 1968).

Boussinesq presented in 1904 an exact solution for the nonlinear differential flow equation by assuming a Depuit–Boussinesq aquifer model with zero water level in the stream and initially curvilinear water table (Hall, 1968). The nonlinear solution yields

$$Q_t = Q_0(1 + a_3 t)^{-2} \tag{6}$$

where a_3 is a constant. Eq. (6) with an additive constant was apparently, according to Boussinesq, first used by Maillet in his low flow analysis of the Vanne River (Brutsaert and Nieber, 1977). Maillet published in 1905 a book in which he demonstrated the applicability of Eqs. (2) and (6) (Hall, 1968). Eq. (6) is also shown by, e.g. Werner and Sundquist (1951), Ishihara and Takagi (1965) and Hornberger et al. (1970) to yield the outflow from an unconfined aquifer, and the equation has found to give a good fit under conditions that correspond well to these (Ambroise, 1988). The curve is a hyperbola and plots as a straight line on a log–log paper for the variables Q_t and $(1 + a_3 t)$.

Werner and Sundquist (1951) obtained, for the outflow from a confined aquifer, Q_t as a sum of n exponential terms:

$$Q_t = Q_0 \sum_{i=1}^{n} b_i \exp(-a_i t) \tag{7}$$

where a_i and b_i are constants. They stated that usually only one term is needed, leading to the simple exponential function (2).

Ishihara and Takagi (1965) expressed the recession flow as a sum of two components, the outflow from a confined aquifer subject to a simple exponential decay (2), and the outflow from an unconfined aquifer expressed as Eq. (6). The unconfined component dominates the lower part of the recession as the rate of recession of the confined component is faster than the unconfined one. Singh and Stall (1971) analysed the outflow from an unconfined aquifer under two boundary conditions; a partially penetrating stream, and a fully penetrating stream where the horizontal impervious layer is assumed at the stream bed. The Boussinesq equation was used to show that the flow follows a single exponential decay only for the first boundary condition. Nutbrown (1975) applied normal-mode analysis to groundwater flow of a partial penetrating stream. Based on the two-dimensional flow equations and the validity of the Depuit assumption, Q_t was expressed as a superposition of many exponential terms:

$$Q_t = \sum_{i=1}^{\infty} A_i K_i^{\,t} \tag{8}$$

where the constants A_i are dependent on the initial value of baseflow, Q_0, and on the original distribution of piezometric heads in the aquifer. Nutbrown and Downing (1976) stated that a single exponential term exists in Eq. (8) when the shape of the piezometric head in the aquifer is relatively smooth and can be represented by a single normal mode. Generally, more than one normal mode is necessary, and the resultant flow is represented by a sum of exponential terms. They further argued that this result is not in conflict with Singh and Stall (1971), as the equations are valid under different boundary conditions of aquifer characteristics.

2.2. Modelling recession as reservoir outflow

The outflow given by Eq. (2) is equivalent to the outflow (Q) from a simple linear storage model with no inflow (I) ($p = 1$ and $K = 1/C$ in Eq. (9)). As the simple exponential equation generally does not satisfactorily represent the recession flow over a wide range of flows, the catchment storage should be given a nonlinear representation or conceptually modelled by more than a single reservoir.

The outflow of a lumped storage model can be characterized by a general function of the type

$$Q = KS^p \tag{9}$$

where S is storage and K and p are constants. The linearity of the storage function can be tested by plotting the hydrograph semilogarithmically (Mitchell, 1972). The storage model is linear only if the recession plots as a straight line. If p is greater than unity, the recession will be concave downward, and if it is less, concave upward. Values less than unity will seldom occur; higher values are, however, typical of many catchments. Eq. (9) with p as a function of S can account for a continually changing relationship between Q and S, i.e.

$$Q = KS^{(1+qS)} \tag{10}$$

where q is a constant.

The storage formulation is only occasionally chosen in recession analysis (Wittenberg, 1993), and the recession process is commonly formulated in terms of the reservoir outflow, which can be calculated given the continuity equation

$$I - Q = \mathrm{d}S/\mathrm{d}t \tag{11}$$

For nonlinear models ($p \neq 1$ in Eq. (9)) and no inflow, the outflow is given by (Brutsaert and Nieber, 1977)

$$Q_t = Q_0(1 + a_4 t)^{p/(1-p)} \tag{12}$$

where a_4 is a constant. Eq. (12) equals the expression in (6) for $p = 2$, which shows that the aquifer modelled by (6) has the same drainage characteristics as a lumped storage model of second order.

Instead of applying a nonlinear drainage equation, the recession curve can be modelled as a combination of linear curves, as the principle of superposition applies at linear solutions. In this way, an aquifer might be linearized by modelling it as a set of n linear reservoirs with a series of recession constants C_i, determined from the equation

$$Q_t = \sum_{i=1}^{n} Q_{0i} \exp(-t/C_i) \tag{13}$$

This equation states that recession flow is a superposition of many distinct exponential terms, and is often referred to as the superposed exponential equation. It is of the same form as the expressions in (7) and (8).

2.3. *Modelling recession as an autoregressive process*

Rewriting Eq. (2b) for $t = 1$ and adding an error term, gives the expression for the first-order autoregressive process, AR(1):

$$Q_{t+1} = kQ_t + e_{t+1} \tag{14}$$

where e_t are assumed independent normally distributed errors with zero mean and constant variance. This model was used by James and Thompson (1970) and Vogel and Kroll (1991) for modelling baseflow recessions. The error term includes both model and measurement errors. Flow measurement errors can generally not be assumed independent of the measured flow, and are commonly expressed in percentage of the flow, a fact that might question the additive error structure of Eq. (14).

Including two recession constants k_1 and k_2, Eq. (14) can be extended to a second-order autoregressive model, AR(2):

$$Q_{t+1} = fQ_t + gQ_{t-1} + e_{t+1} \tag{15}$$

where $f = k_1 + k_2$ and $g = -k_1k_2$. Estimates of the recession parameters of Eqs. (14) and (15) can be obtained by a least-squares method. If the errors are assumed independent a direct solution can be obtained. James and Thompson (1970) presented an iterative weighted least-squares procedure assuming correlated errors, where the weights are functions of the parameters to be estimated.

Spolia and Chander (1974) formulated in general terms the structural relationship between the parameters of an ARMA (autoregressive moving average) model and a conceptual formulation of n linear reservoirs in series. For two linear reservoirs, the ARMA model reduces to the AR(2) model presented in Eq. (15).

2.4. Empirical relationships

The applicability of Eq. (2) has also been demonstrated in many empirical studies, and it is today the most widely used equation in recession analysis. Still, often more complex equations are sought to describe the recession curve for a wide range of flows. One simple alternative to Eq. (2) is achieved by adding a constant:

$$Q_t = (Q_0 - b)\exp(-t/C) + b \tag{16}$$

The discharge, Q_t, in Eq. (16) will asymptotically approach the constant b for large t. The equation is referred to as the 'ice melt' exponential by Toebes and Strang (1964), and suggested for use in areas with snow and ice present. Radczuk and Szarska (1989) applied Eq. (16) to rivers in Poland, and interpreted b as the limit baseflow. Clausen (1992) modelled the recession flow of two Danish streams, both fed by two main aquifers, and found Eq. (16) to be superior to the simple exponential Eq. (2) and similar in performance to the more complex Eq. (8) consisting of two terms and an additive constant.

Otnes (1953) studied recessions from catchments in southern Norway, and suggested an hyperbola of the form

$$Q_t = a_5 t^{-1} - Q_0 \tag{17}$$

where a_5 is a constant. Based on an empirical study Otnes later found that the equation (Otnes, 1978)

$$Q_t = a_6 t^{-r} \tag{18}$$

where a_6 and r are constants ($r > 1$), was generally suitable for Norwegian catchments, which are characterized by a high lake percentage. Until recently, recession studies in Norway have commonly been based on Eq. (18) (Gjørsvik, 1970;

Andersen, 1972; Tjomsland et al., 1978). The equation is similar to the ice melt hyperbola presented by Toebes and Strang (1964):

$$Q_t = a_7 t^{-r} + b \tag{19}$$

where a_7 and b are constants, b representing the asymptotical discharge of the curve. This expression was similar to (16) suggested for use in areas with snow and ice. Expression (18) plots as a straight line with slope $-r$ on a log–log graph of Q_t and t. A similar straight-line relationship is valid for Eq. (19) if Q_t is replaced by $(Q_t - b)$ and provided a suitable b has been chosen.

Expressions with only one constant are preferable in catchment comparative studies, as it can be difficult to interpret the relative importance of two or more parameters. On the other hand, expressions with more than one parameter generally give a better fit to the flow data and should be used for more detailed catchment studies.

Eqs. (7), (8) and (13) express the recession curve as a combination of linear curves. Several workers have found empirically that the recessions are adequately represented by a few components of these equations; Eq. (13) with $n = 2$, was discussed by Boussinesq (1904) for modelling nonlinear recession curves, by Barnes (1939) with $n = 3$, for the separation of the flow into three linear components, and more recently by Pereira and Keller (1982a) and Petras (1986) for $n = 2$ and 3.

3. Derivation of a characteristic recession

In a humid climate, rainfall frequently interrupts the recession period. As a result, each discharge series produces a series of recession segments of varying durations. Several methods have evolved to construct a master recession curve from the selection of shorter recessions. A major problem is the high variability encountered in the recession behaviour of individual segments. The segments represent different stages in the outflow process, and a physically based short-term or seasonal variation in the recession behaviour adds to the problem of deriving a characteristic recession.

The master recession methods try to overcome the problem by constructing a mean recession curve. Information on the recession variability is then lost. If the variability within the catchment is of the same order as between catchment variation, care should be taken when comparing master recession curves. The variability can be estimated by obtaining a quantitative expression for each segment. If we assume there are n recession segments, then the p recession characteristics, C_p, can be obtained either from the master recession (A) or for each segment separately (B), as illustrated in Fig. 2. In the latter case, a mean value can be calculated to represent average catchment conditions. The method also allows the variability to be given a statistical consideration, and if of interest, possible causes behind the variation can be investigated.

If the recession curve is represented by the simple exponential equation, there is only one constant to be estimated, $p = 1$. The lack of fit of the exponential equation for a wide range of flows has encouraged the use of a superposed exponential equation. Hydrograph separation procedures have commonly been used to define

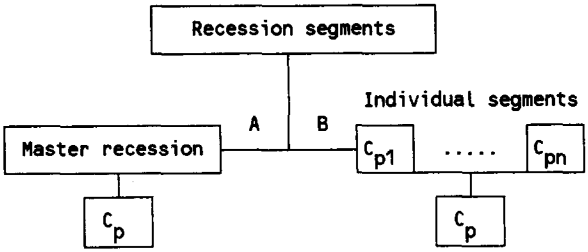

Fig. 2. The recession characteristics, C_p, can be estimated from a master recession curve (A) or from an ensemble of recession segments (B).

the separate exponential terms, and the different components interpreted as indicators of different flow components of the recessions.

3.1. Hydrograph analysis

Interpreting recession flow using graphical techniques, as first introduced by Barnes (1939), has frequently been applied to distinguish between different flow components (Laurenson, 1961; Kunkle, 1962; Hewlett and Hibbert, 1963; Ineson and Downing, 1964; Singh and Stall, 1971; Klaassen and Pilgrim, 1975; Boughton and Freebairn, 1985) or for separation of the total hydrograph (Linsley et al., 1958; Kulandaiswamy and Seetharaman, 1969; Bates and Davies, 1988). The recession curve has traditionally been separated into the linear components of surface, unsaturated and saturated flow. The components are thought to represent different flow paths in the catchment, each characterized by different residence times, the outflow rate of groundwater flow from a catchment being lower than the recession rate of the other flow components.

A change in recession rate has likewise been ascribed to the heterogeneity of an aquifer or the presence of widely different aquifers (Riggs, 1964; Petras, 1986), whereas the single recession curve as expressed in Eq. (2) is thought to represent homogeneous conditions in terms of transmissivity and storage properties.

The recession curve is in either case considered a composite curve, which has generally been modelled by the superposed exponential Eq. (13). Each term is thought to represent a different flow component, and has traditionally been determined by graphical analysis (Singh, 1988). If each term dominates at specific times, $\log Q_t$ will plot as a series of straight lines (Klaassen and Pilgrim, 1975).

Many graphical separation methods employed are highly subjective and are of limited use as indicators of the flow processes operating (Nash, 1966; Hewlett and Hibbert, 1967; Anderson and Burt, 1980). In more recent works, subjective methods for analysing compound recession curves are abandoned in favour of analytic separation procedures (James and Thompson, 1970; Browne, 1978; Pereira and Keller, 1982a). The hydrograph separation methods presented by these workers focus on baseflow recessions, and do not attempt to model a continuous separation.

As opposed to graphical methods, isotopic and chemical hydrograph separation techniques can be applied as a means of identifying the sources and pathways of river runoff (Sklash, 1990; Neal and Hornung, 1990). O'Connor (1976) found that the time of detention of chemical concentrations in ground and surface water could be approximated by the recession constants of the surface and groundwater components of the hydrograph. The recession constants were obtained from traditionally semilogarithmic plotting (Linsley et al., 1958). Together with direct measurements of the different flow paths in the field, chemical parameters and environmental isotopes have the potential to provide valuable information on the sources of flow during recession periods and thus identify the causes behind the high time variability in recessions found in some catchments.

3.2. Recession selection algorithms

In the case of a single linear aquifer the obtained values of the recession constant are independent of the flow or time origin, and the choice of recession segment is irrelevant. In practice, this is rarely the case, and the time origin of the recessions cannot be chosen arbitrarily. It has proved difficult to find a consistent way of selecting recessions from a continuous flow record. Flow data alone or in combination with precipitation records are used as a first step to define the recession periods. A recession period lasts as long as the streamflow does not rise, or precipitation above a given limit occurs. The recession segments are then selected from the set of recession periods (Fig. 1), and defined by a starting and duration criteria. Both these criteria have been given numerous representations in the literature. This section briefly presents some recession selection algorithms encountered.

The start of a recession segment, often called initial discharge, can take the form of a constant value or be a variable. It can be represented by one or more values for a catchment, depending on the number of flow components considered. A variable starting value is a function of the particular method adopted for selecting recession segments, whereas a constant is a fixed value determined prior to the analysis.

A constant value restricts the recession analysis to the range of flow below a given discharge (Institute of Hydrology, 1980; Demuth, 1989; Tallaksen, 1989; Gottschalk and Perzyna, 1993). This criterion can be used alone or in combination with other restrictions. As the discharge value is a variable related to the degree of catchment wetness, one might assume initial conditions of catchment wetness to be similar at a given discharge.

A variable starting value can be defined as the discharge at a given time after rainfall or after peak of discharge, and will take on different values for each event. The first part of a recession period is normally left out to reduce the influence of surface flow, and sometimes also the last to prevent the influence of the next storm. The number of observations to discard should be selected based on knowledge of the typical response time of the catchment. Variable starting levels have also been determined from hydrograph analysis, traditionally as introduced by Barnes (1939), or more recently by automatic methods as suggested by Singh and Stall (1971) and

Browne (1978). Vogel and Kroll (1991) defined the recession period to start when a 3 day moving average began to decrease and to end when it started to increase.

The recession length, represented by the number of timesteps in the flow sequence, can, similar to the start of recession, be a constant or variable value. A minimum length of recession is usually chosen between 4 and 10 days, depending on the mean duration of dry spells in the region. Recession periods above 10 days have been studied in work by Ando et al. (1986) and Vogel and Kroll (1991). Tallaksen (1989) suggested selection of standard recessions, where both initial discharge and recession length are constants, to reduce the variability in recession characteristics that could be ascribed to limitations in the simple exponential equation applied, and to permit an evaluation of the influence of natural factors on the variability.

3.3. *Master recessions*

Traditionally, graphical methods have been used to construct a master recession curve (Toebes et al., 1969), and the two most commonly used methods are the matching strip and the correlation method. Another frequently used method is the tabulating method, but as pointed out by Hall (1968), these are all three somewhat arbitrary hydrograph methods. In the tabulating method (Johnson and Dils, 1956), which is rarely used nowadays, the recession periods are tabulated, shifted until the discharges agree approximately, and mean discharges are calculated for each timestep in the period. In the matching strip method (Toebes and Strang, 1964), individual recession segments are plotted and adjusted horizontally until they overlap in the main parts. The master recession curve is then constructed as the mean line by best eye fit through the set of common lines. This method permits a visual control of irregularities in the recession curve, but often telescopes or contracts the true recession.

Nathan and McMahon (1990) presented a semiautomated procedure that automatically selects the recession periods and plots them in descending order on semilogarithmic scales. The operator then interactively shifts the recessions along the ordinate axis until they overlap in the desired fashion. This procedure has made the matching strip method less time consuming; however, it is still subjective. Wingård (1976), Petras (1986) and Demuth (1989) presented graphical shifting procedures for the recession segments which allows the master curve to be constructed without any subjective judgements.

In the correlation method (Langbein, 1938), the discharge at one time (Q_t) is plotted against discharge one time interval later ($Q_{t+\mathrm{d}t}$) for the selected recession periods, and a curve fitted to the data points. If the recession rate follows an exponential decay as expressed by Eq. (2b), $Q_{t+\mathrm{d}t}$ vs. Q_t will plot as a straight line with slope k, where

$$k = (Q_{t+\mathrm{d}t}/Q_t) \qquad (20)$$

Federer (1973) calculated k for 1 day segments and grouped the values by date and streamflow rate at noon. The mean value for each group was calculated, and seasonal, average recession curves constructed. The method was later applied by Grip (1977) and Brandesten (1988).

The uncertainties involved in defining recession periods from the hydrograph have made some workers prefer the concept of the correlation method which eliminates the variable t from the analysis and instead expresses the rate of change in flow for a differential dt. The rate of change can generally be expressed as a function of Q:

$$\mathrm{d}Q/\mathrm{d}t = f(Q) \tag{21}$$

where f is a characteristic function for a given catchment. The equation can for flow data be approximated by differences and the function $f(Q)$, representing the master recession, determined from a plot of $\mathrm{d}Q/\mathrm{d}t$ against Q. The recession hydrograph itself can be derived by integrating $f(Q)$. Jones and McGilchrist (1978) chose instead to model the rate of change in river height ($\mathrm{d}h/\mathrm{d}t$) as a function of h.

The choice of analytical expression to be fitted to the correlation plot is based on the same considerations as for the observed hydrograph. For the nonlinear flow equation in (12), (21) takes the form of a power function:

$$\mathrm{d}Q/\mathrm{d}t = -cQ^d \tag{22}$$

where c and d are constants which can be expressed in terms of the constants Q_0, a_4 and p in Eq. (12). For a simplified linear equation as given in Eq. (2), that is $d = 1$, Eq. 22 reduces to

$$\mathrm{d}Q/\mathrm{d}t = -cQ \tag{23}$$

which in terms of differences can be expressed as

$$(Q_{t-1} - Q_t)/\Delta t = -c(Q_{t-1} + Q_t)/2 \tag{24}$$

The graphical analysis of $Q_{t+\mathrm{d}t}$ against Q_t or the equivalent form of $\mathrm{d}Q/\mathrm{d}t$ against Q, is often performed by means of the upper and lower envelope of points, representing the maximal and minimal observed recession rates, respectively (Knisel, 1963; Brutsaert and Nieber, 1977; Browne, 1978; Zecharias and Brutsaert, 1988; Troch et al., 1993). The function f determined from the envelope curves can then be defined as the master recession.

Analysis of the correlation plot requires high accuracy in the low flow measurements, and it is important that the time interval is selected in accordance with the quality of the flow data. Daily time intervals are frequently used, but have often shown to be insufficient for the data in question. The Institute of Hydrology (1980) argued for an interval of 2 days, as a compromise between shorter periods of low accuracy, and longer periods which tend to eliminate many recession periods from consideration. Nathan and McMahon (1990) stressed the importance of selecting as long a time interval as possible when using the correlation method, owing to the subjective element involved when fitting a curve to the data points. They further stressed that a small range in slope represents a significant proportion of the range of baseflows, so the uncertainty involved in applying this method might be considerable.

Brutsaert and Nieber (1977) determined graphically the function f of the lower envelope of data points. Over most of the range of Q the slopes of the lower envelopes were reasonable constant, with d in Eq. (22) close to 1.5, which they showed to be in accordance with the nonlinear Depuit–Boussinesq model. For higher flow rates, when the free water surface is no longer curvilinear, d was found to be in the order of three, in agreement with applying the Boltzmann similarity to Boussinesq's equation. These theoretical values for d were also found by Troch et al. (1993) to be in good agreement with observed recession rates.

The subjective element of curve fitting can be avoided by considering all the flow data in the correlation plot, not only the lower envelope of points. A straight line can then be fitted by linear regression, and d estimated as the slope of the line. Vogel and Kroll (1992) found d to be close to unity in a study of 23 catchments in Massachusetts, whereas a value close to 1.5 was obtained by Troch et al. (1993) for a Belgian catchment.

3.4. Individual recessions

The master recession methods try to overcome the problem of time variability in recessions by constructing a mean curve (Fig. 2). However, it is possible to leave the master recession principle and instead calculate a quantitative expression for each segment (Chidley, 1969; Klaassen and Pilgrim, 1975; Tallaksen, 1987; Vogel and Kroll, 1991; Gottschalk and Perzyna, 1989; Clausen, 1992).

Average recession characteristics can be obtained from the set of n recession curves. Assuming n groups of observations, each with j observations (x_{ij}, y_{ij}), $i = 1, \ldots, n$, $j = 1, \ldots, m$. If a separate linear regression line can be fitted to each group of observations by the least-squares method, and the sum of squares for variation about each line obtained, four lines can be considered (Brownlee, 1960): (i) the n individual lines; (ii) the n parallel lines with an average slope k; (iii) the least-squares line for the group means (x_i – mean, y_i – mean); (iv) the overall regression line which assumes that all observations come from a single population.

Brownlee presented an expression for the average slope in (ii), which adjusts for differences in the number of observations, j, in each group. The average slope can be calculated if the null hypothesis, which assumes that there is not a significant difference between the n slopes, is accepted. This approach was used by Bako and Hunt (1988) to derive a mean baseflow recession constant. It is necessary to account for the number of observations in each flow sequence to obtain unbiased estimates of mean recession characteristics (James and Thompson, 1970; Pereira and Keller, 1982b). If the number of observations (j) in each group is equal, however, the average slope will equal the arithmetic mean of the n individual slopes.

4. Optimization of recession parameters

Despite the form of the recession plot, as a hydrograph segment or a correlation plot, a functional relationship has to be fitted to the data in order to obtain a quantitative expression. The recession model can be chosen amongst analytical expressions selected a priori from theoretical considerations, or determined from empirical judgements as discussed in the preceding sections.

Traditionally, the selected recession equation has been fitted to a set of points by a simply subjective best eye fit of the data, as in the matching strip method. Such a procedure is subjective and liable to human error. In contrast to manual techniques, an automatic method is fast, reliable and objective, and should be used both to obtain a characteristic recession and to estimate its recession parameters.

An automatic estimation of recession parameters is commonly performed using a simple or weighted least-squares regression for determining the functional relationship of the recession plot (James and Thompson, 1970; Pereira and Keller, 1982b; Tallaksen, 1987; Bako and Hunt, 1988; Perzyna, 1990; Clausen, 1992; Vogel and Kroll, 1992; Troch et al., 1993). Jones and McGilchrist (1978) estimated the parameters of the regression model by the method of maximum likelihood. Automatic methods generally allow for statistical considerations of the model fit and the estimated parameters. Vogel and Kroll (1991) pointed to the importance of obtaining minimum variance and unbiased estimates of the recession constant, and Pereira and Keller (1982b) developed a curve-fitting procedure which particularly focuses on reduced variance of the residuals.

Yates and Snyder (1975) suggested modelling of the recessions through convolution, owing to the problems of finding an analytic expression to fit the recessions. The six parameters of the model are determined by optimization with streamflow records using a nonlinear least-squares procedure. The discharges are averaged for several recessions, and the method is suitable for predicting mean recession behaviour (Yates and Snyder, 1975; Ratomska and Partyka, 1986).

5. Time variability in recessions

A considerable variation in the recession rate has been found for most catchments. It is important that we understand and evaluate, preferable quantitatively, the effect of different factors on the recession rate. The variation depends on physical factors, but also on the particular recession model and calculation procedure chosen. It is important to minimize the influence of the latter when evaluating natural physical causes behind the variation.

Limitations in the selected recession equation to model a wide range of flows, will impose variability in the calculated characteristics, and any subjective judgements in the method for deriving a characteristic recession will influence the result. Tallaksen (1991) found in a Norwegian study that the recession constant calculated from the simple exponential equation depended both on start and length of the recession segment. In many small catchments a special problem arises as records do not exhibit a high measurement precision for low flows, and several days may pass before a lower value is recorded. The ensuing steps in the discharge series impose difficulties in the selection and modelling of recessions.

Physically based variation in the recession rate is caused by difference in climate during the time of recession, but is also determined by the conditions prevailing prior to the start of the recession. The amount of water stored in the catchment depends on previous weather conditions and type of aquifers. In larger catchments spatial variability in the storm patterns causes differences in the distribution of flow paths in the catchment, which in turn will influence the drainage pattern (Laurenson, 1961). The presence of several aquifers also contributes to the variability found in recessions. Both areal and vertical differences in aquifer characteristics cause this complexity, and a seasonal change in the proportion of discharge provided by different aquifers might be found.

Climate influences the recession rate during the time of recession by recharge from precipitation or snow melt, or by losses from evapotranspiration. Diurnal fluctuation of streamflow is a result of water being lost by evapotranspiration during daytime at a higher rate than at night, and is a well-known effect of riparian transpiration. Normally a pronounced increase in the diurnal fluctuations is observed during the warm season (Troxell, 1936; Croft, 1948; Tschinkel, 1963; Reigner, 1966).

Several workers have recognized a seasonal variation in the recession behaviour which follows the seasonal change in evapotranspiration (Croft, 1948; Knisel, 1963; Singh and Stall, 1971; Federer, 1973; Trainer and Watkins, 1974; Grip, 1977; Ando et al., 1986; Ambroise, 1988; Brandesten, 1988). During warm weather and especially in the active growing season, water lost by evapotranspiration has a marked influence on low flows and recession dynamics. Steeper recession curves are generally found during the warm season, along with a reduction in baseflow. This is particularly noticeable in areas with shallow groundwater tables and extensive vegetation, where a drying of the upper soil layer can be succeeded by capillary transport from the groundwater.

Some studies have approached the problem of predicting the amount of water lost by evapotranspiration. The loss has been estimated by constructing a 'potential' recession curve, which is thought to represent time of little or no evapotranspiration (Farvolden, 1963; Tschinkel, 1963; Reigner, 1966; Weisman, 1977). The evapotranspiration loss is assumed to be related to the difference between the actual and the potential recession curve. The method poses problems in defining the potential recession curve.

Tallaksen (1989) included the recession mean value and coefficient of variation (CV) calculated from individual segments of equal lengths, in a regional study of 68 Norwegian catchments. It was not possible to relate CV to any of the flow, catchment or climate characteristics included. This suggests that the variance is independent of the low flow regime, a fact also indicated by the lack of correlation with the mean recession value. The study revealed a high variability in the recession constant, and in a later study (Tallaksen, 1991) it was found that this could partly be explained by a seasonal variation in evapotranspiration.

6. Concluding remarks

The recession curve contains the integrated information of how different factors influence the outflow process. To take full advantage of this information and make possible a wider use of recession analysis, it is necessary to improve further the methods at present applied in recession analysis. Most important in this context is to incorporate in a quantitative manner the high variability encountered in the recession behaviour of individual segments. Variability owing to model limitations and calculation procedure should be minimized, whereas physically based short-term or seasonal variations should be accounted for in agreement with the purpose of the study.

In general, the methods selected for obtaining a characteristic recession and to estimate its recession parameters should be fast and objective, to ensure consistency in the derivation and applicability for larger data sets. This is essential in regional studies if intercomparison is to be meaningful. The high time variability in the recession curve argues against the use of a master recession curve except as an overall approximation which might be applicable for use at the regional scale. When selecting an expression to fit a recession segment, one has to consider which part of the curve is most important. In a regional study where the recession characteristics are thought to represent catchment storage properties, it is the speed with which the low flow is reached that is important. For regional analysis, a simple expression is preferable as the recession behaviour will vary considerable between the catchments.

Low flow forecasting is concerned with the lower end of the recession curve, and it is necessary to analyse the recession characteristics in a detailed manner where more complicated solutions might be sought. The time variability found in recessions should be incorporated in any low flow forecast, merely as a statistical variance or given as a conditional forecast with reduced variability.

A seasonal form of the recession curve is a result of the flow being more affected by evapotranspiration in the warm season. The observed seasonal difference in recession dynamics demonstrates generally faster recession rates in summer than in autumn or winter. The question is to what extent this can be explained by higher evapotranspiration losses during the time of recession, or whether the effect high losses have on the general catchment wetness on a longer time-scale is more important. The drainage pattern following a summer storm on initially dry soils will differ from a typically autumn storm when soils are well saturated. Great heterogeneity in soil and geology within a catchment will enlarge this difference. Further research is required on this topic, and a better understanding of the physical processes governing recession flow and its variation in time is necessary for advances to be made in recession analysis.

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